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Easing the modeling of lossy lines

SIMPLE MEASUREMENTS AND STRAIGHTFORWARD TECHNIQUES NOT ONLY CAN OFTEN OBVIATE THE NEED FOR EXPENSIVE SIMULATION TOOLS, BUT ALSO CAN PROVIDE A MORE INTUITIVE FEEL FOR NETWORK BEHAVIOR.

With common data rates quickly entering the gigabit-per-second domain, frequency-dependent losses in interconnects—cables and pc-board traces—are quickly becoming major obstacles to further speed increases. Thus, you can no longer ignore transmission losses; during system development as well as for device and system test, timing-budget planning requires accurate modeling of these losses.

Today, you can buy a variety of software—both for RF- and time-domain (usually digital) applications—to predict those losses and to build equivalent-circuit models based either on theoretical calculations or on actual measurements. Although powerful for experienced engineers, these tools have two severe downsides for engineers who need only occasionally to build usable models with a minimum of effort: First, the tools are expensive, making it difficult to justify their cost. Second, the more powerful their features, the more arduous their learning curve, so designers cannot be productive with them without spending a lot of time using them—again an important obstacle for an engineer whose main task is something other than transmission-path modeling.

This article describes an easy-to-follow measurement-based method for creating practical models for lossy cables and pc traces, using nothing but easily available and inexpensive Spice and Excel software tools. As an added advantage, you will gain insight into the effect and behavior of those losses. In the end, it is more rewarding to solve a problem in a pedestrian way than to plug data into a program and then mindlessly believe the results it produces.

LOSSY TRANSMISSION LINES

Figure 1 shows the general model of a lossy transmission line: Its components are the series inductance L, shunt capacitance C, series resistance R, and shunt conductance G. For a homogeneous transmission line, those parameters are distributed evenly along the length of the line. For an ideal lossless transmission line, R and G are zero. (Note that R is a resistance, which you measure in ohms, and G is a conductance, which you measure in siemens=1/ohms.) A frequency-dependent R_F models ohmic resistance and skin-effect loss. The skin-effect loss is nothing other than ohmic resistance aggravated by the inhomogeneous current distribution that results from the skin effect. Frequency-dependent G_F represents dielectric losses. This frequency dependency causes all of the trouble in modeling signal propagation. The general expression for the characteristic impedance, Z_0, of this line is:

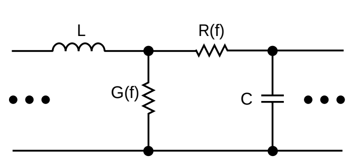

Figure 1 In the general model of a lossy transmission line, its components are the series inductance L, shunt capacitance C, series resistance R, and shunt conductance G.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \tag{1}$$

and $\omega=2\pi f$ (references 1 and 2). For sufficiently high frequencies, relatively small losses, or both ($R \ll \omega L$, and $G \ll \omega C$), you can approximate this expression as:

$$Z_0 = \sqrt{\frac{L}{C}}. \tag{2}$$

This equation is the same as that for a lossless transmission line. Practical digital interfaces all fall into this range. This piece of information is important because it tells you that—apart from some signal attenuation—the signal propagation is the same as for lossless lines. For example, Z_0 is largely independent of frequency, and the following expressions hold:

$$T_{PD} = \sqrt{LC}; \quad L = T_{PD} \times Z_0; \ \text{and} \quad C = T_{PD} / Z_0. \tag{3}$$

Here, T_{PD} is the propagation time through the line (Reference 1). This set of equations allows you to determine model parameters—in this case, L and C—based on measurements of propagation time and characteristic impedance. The general propagation constant of a lossy line is:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L) \times (G + j\omega C)}, \tag{4}$$

where α (the real-valued part) describes the signal attenuation, and $j\beta$ (the imaginary part) describes the wave propagation along the line. Under the same low-loss assumptions as before, Equation 4 approximates to:

$$\alpha \approx \frac{1}{2} \times \left(\frac{R}{Z_0} + GZ_0 \right); \quad \beta \approx \omega \times \sqrt{LC} = \omega \times T_{PD}. \tag{5}$$

For a lossless line, α is zero. An important conclusion from α in Equation 5 is that the signal gain through a lossy line is:

$$\text{GAIN} = \frac{V_{OUT}}{V_{IN}} = e^{-\frac{R}{2Z_0}} \times e^{-\frac{GZ_0}{2}}, \quad (6)$$

where V_{IN} and V_{OUT} are the signal amplitude entering and exiting the line, respectively. Although transmission theory usually talks in terms of gain, in this case, the gain is always less than one, so it is really a loss. **Equation 6** is the third formula you need to build a model, because it relates the measured loss through a cable or trace to the loss parameters R and G. Because gain or loss can span a wide range of magnitudes, it is usually stated in the logarithmic decibel scale. Thus,

$$\text{GAIN}_{dB} = 20 \times \log(\text{GAIN}) = -20 \times \log(e) \times \left(\frac{R}{2Z_0} + \frac{GZ_0}{2} \right);$$
$$\text{LOSS}_{dB} = -\text{GAIN}_{dB}. \quad (7)$$

LOSS MODELS

The only piece of theory still missing is a set of models for the behavior of the different loss contributors versus frequency. The easiest is ohmic dc loss, because it does not depend on frequency. It forms one part of the series resistance R, and is denoted as R_{DC}. Skin effect causes the second part of the series resistance. For a perfect coaxial cable, the skin resistance is proportional to the square root of the frequency, and you can even derive an analytical expression for the proportionality factor (**references 1** and **2**). For arbitrarily shaped transmission lines, such as striplines in a pc board in which the field distribution is more complicated, this approach no longer works, but the square-root behavior remains at least approximately valid. Thus, you can model the series resistance due to skin effect as:

$$R_{SKIN} = k_{SKIN} \times \sqrt{f}. \quad (8)$$

For these purposes, the factor k_{SKIN} is simply a fit parameter, which you adjust to match the measured behavior. The skin resistance and the dc resistance add to the total resistance to produce:

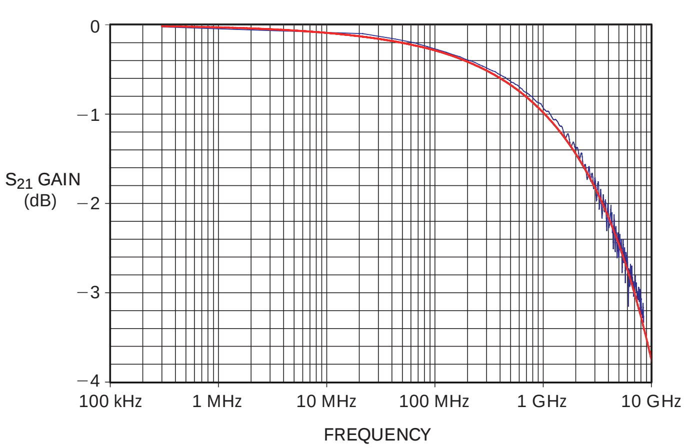

Figure 2 In this comparison of the measured-loss curve and the model-simulation results, the match is excellent over the whole range, especially considering that the model comprises just one circuit element in PSpice.

$$R = R_{DC} + R_{SKIN}. \quad (9)$$

This **equation** is only a rather crude approximation, but, as you will see later, it greatly simplifies fitting the loss behavior to the measured data. **Reference 2** mentions that you can achieve a better fit to the actual resistance trend by taking the root-mean-square of R_{DC} and R_{SKIN}—that is, $R=\sqrt{(R_{DC}^2 + R_{SKIN}^2)}$.

Finally, dielectric losses—represented by the shunt conductance, G—increase approximately linearly with frequency. This rule is just an approximation, because, for any real-world dielectric material, the loss tangent, tanδ, varies somewhat with frequency, although the variation is much smaller than the frequency variation. That is,

$$G = 2\pi \times f \times C \times \tan\delta = k_{DIEL} \times f = k_{DIEL} \times \left(\sqrt{f}\right)^2. \quad (10)$$

You'll soon see the advantage of the strange square-root formulation. Just as for the skin effect—because you usually don't know the effective loss tangent—k_{DIEL} is a fit parameter that you adjust to best fit the measurement.

You can now insert those loss models into the general expression for the total line losses:

$$\text{GAIN}_{dB}(f) =$$
$$-20 \times \log(e) \times \left(\frac{R_{DC}}{2Z_0} + \frac{k_{SKIN}}{2Z_0} \times \sqrt{f} + \frac{Z_0 k_{DIEL}}{2} \times \left(\sqrt{f}\right)^2 \right). \quad (11)$$

If you look closely, you see that—if you plot the measured gain in decibels versus the square root of the frequency instead of the frequency itself—the equation reduces to a simple parabola:

$$\text{GAIN}_{dB}(x) = a + bx + dx^2.$$
$$x = \sqrt{f}.$$
$$a = -20 \times \log(e) \times \frac{1}{2Z_0} \times R_{DC}. \quad (12)$$
$$b = -20 \times \log(e) \times \frac{1}{2Z_0} \times k_{SKIN}.$$
$$d = -20 \times \log(e) \times \frac{Z_0}{2} \times k_{DIEL}.$$

In other words, once you have measured the loss in decibels over a range of frequencies, the actual data fitting becomes almost trivial. One possibility is to use Excel for this task, because almost every engineer has it readily available, and it can perform polynomial fits. But any other fitting software will also do the job. From the fit parameters a, b, and d, you can then easily calculate the loss parameters R_{DC}, k_{SKIN}, and k_{DIEL}.

MEASUREMENT AND MODEL-BUILDING

For practical measurements, a VNA (vector-network analyzer) is the tool of choice. A scalar network analyzer will do just fine for the loss measurement, but you will need to determine the propagation delay, T_{PD}, with some alternative method, such as TDR (time-domain reflectometry). If no network analyzer is available, you can measure the attenuation using a sinewave source (an RF generator) and an oscilloscope, though it is a bit tedious to acquire a sufficient number of points, and accurate measurement of small losses is difficult. The reasons that it is better to do the modeling in the frequency domain are twofold: First, losses have an easy-to-describe behavior in the frequency domain, in contrast with rather difficult-to-interpret time-domain behavior (for example, step response). Second, VNAs have unparalleled amplitude-measurement—and, therefore, loss—resolution, which can cover the whole range, from very small to very large losses. (A VNA can easily reach an SNR of 100 dB—a factor of 10^5, whereas an oscilloscope is hard-pressed to reach even 60 dB—a factor of 10^3.) Nevertheless, the resulting Spice model is not restricted to frequency-domain simulations (ac sweeps). Rather, it performs just as well in a time-domain simulation (transient response).

You need to set the frequency sweep to logarithmic and measure the absolute value of the transmission coefficient, S_{21}—plotted on a decibel scale. S_{21} is then identical to the gain in decibels from before. (Note that S_{21} is a complex value; it has both magnitude and phase.) You then transfer the data to a computer equipped with Excel or some other plotting program, replot the curve versus the square root of the frequency, and use Equation 12 to fit the curve to the data and to obtain R_{DC}, k_{SKIN}, and k_{DIEL}. The choice of frequency sweep—linear or logarithmic—has a slight influence on the fit result: A linear sweep overemphasizes the high-frequency range, whereas a logarithmic sweep distributes the measured data points evenly over the whole range.

Next, change the VNA setup to display the group delay of S_{21}. The group delay is defined as the derivative of the phase, ϕ, over the frequency, f. That is,

$$T_G = \frac{d\varphi}{df}. \quad (13)$$

For dispersionless paths, the group delay has a simple meaning; in those cases, it is identical to the propagation delay, T_{PD}, of the path. Fortunately, this scenario normally closely approximates the real situation. The condition for constant group delay is that the dielectric constant, ϵ, does not vary with frequency. The dielectric constant can be truly frequency-independent only

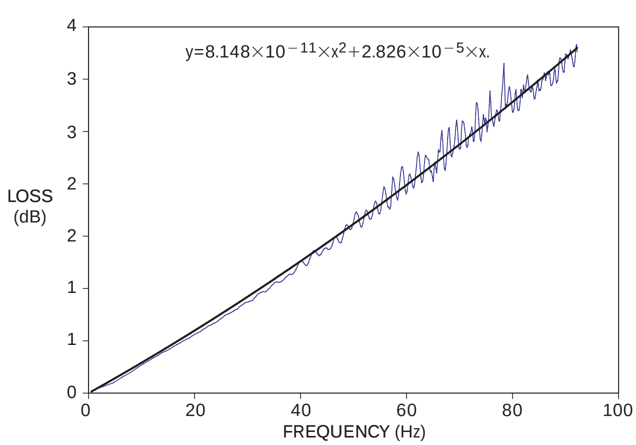

Figure 3 In the parabolic fit and the resulting fit parameters, the dc resistance is negligible; so, in the fit, the y intercept is set to zero.

TO MINIMIZE THE IMPACT OF FRINGE EFFECTS, ALWAYS START WITH THE LONGEST CABLE OR TRACE AVAILABLE.

for lossless media; whenever losses occur, the dielectric constant changes over frequency (governed by the so-called Kramers-Kronig relation). (If the VNA does not offer the option to display the group delay directly, you can instead display the phase. You then calculate the group delay from the slope of the phase curve.) One potential trap is that the VNA calculates the group delay numerically, based on the phase difference between adjacent data points. If the frequency spacing between points is too large, this approach can yield erroneous results because the phase wraps around every 360°. To avoid this problem, you need to ensure that the VNA sweep uses sufficiently closely spaced points. A simple test is to double the number of points and verify that the displayed group-delay curve does not change. The group-delay display usually yields useful readings only in the region above 100 MHz; for lower frequencies, the phase change is small because the wavelength is much longer than the propagation time, and measurement noise and other inaccuracies thus heavily impact the result. You usually know the path impedance, Z_0, from pc-board design parameters or the cable data sheet. In digital applications, it is almost without exception 50Ω. Thus, you can calculate the total line capacitance, C, and the total line inductance, L, using Equation 3.

A few more measurement hints may be useful. First, to minimize the impact of fringe effects, always start with the longest cable or trace available. In other words, path loss must dominate effects of impedance mismatches at the connection points at both ends of the cable or trace—for example, SMA connectors or probes. Otherwise, characterization and mathematical de-embedding of those connectors, which go far beyond the scope of this article, become necessary. This area is one in which professional modeling tools are useful. Second, use the widest frequency range available—as low and as high as the VNA can go—because the wider the range, the more reliable the curve fit will be. It is always more accurate and reliable to interpolate than to extrapolate, although one of the benefits of this fitting method is that it allows you to confidently extrapolate losses beyond the measured range if necessary. Ideally, you should go to at least twice the highest frequency present in the signal—that is, to at least twice the signal bandwidth. For a digital signal, the signal's 10 to 90% rise time, rather than the data rate or clock frequency, gives the approximate bandwidth:

$$BW \approx \frac{0.33}{T_{R,10/90}}. \quad (14)$$

Spice is a popular simulation tool. Among Spice variants, Orcad's PSpice is a good choice because it offers a lossy-transmission-line model as a built-in library component, and a free, fully functional, downloadable demo version is available that restricts only the number of components you can use (Reference 3). The fact that the download is free greatly reduces the cost of getting started if you want to try modeling. The only issue is that PSpice does not offer frequency as a parameter for lossy lines but instead provides the Laplace parameter, s=2πjf. You can quickly overcome this limitation by substituting:

$$f = \text{abs}\left(\frac{s}{2\pi}\right), \qquad (15)$$

In addition to the parameters C, L, R, and G, the lossy-transmission-line model in PSpice offers the LEN (length) parameter. Because the calculated parameters refer to the total line, you can simply set LEN to 1. After you model the line, this parameter provides a way to scale the model up or down for different line lengths—for example, you have measured a long line to minimize connector effects but want to build a model for a shorter section of the same type of line. Finally, if you want to use formulas for parameters in PSpice, you must enclose them in braces.

If you have no access to the far end of the line because, for example, it ends in a socket or in a needle-probe head, you can't make transmission measurements. The easy solution is to measure the reflected signal, S_{11}, instead of S_{21}, because doing so requires only one connection to one end of the line. For this approach to work, the line's other end—that is, the far end—must remain unterminated. The signal then traverses the line, is fully reflected at the far end, and returns to the source, so it effectively traverses the line twice. Data collection and fitting occur as usual. The only difference is that you must halve all measured values to represent a single traversal. This procedure neglects the effect of the parasitic fringe capacitance that can differ when the line is open compared with when it is terminated at the far end.

AN EXAMPLE

The following example applies the theory to a practical situation. The object to be modeled is a coaxial cable with an SMA connector on each end. The intended signal bandwidth is approximately 2 GHz. The VNA's frequency range—300 kHz and 8.5 GHz—is sufficient for this application. You need to take care in calibration, however, because even slight errors affect the accuracy of the measured parameters, especially in the low-frequency region in which the losses are small.

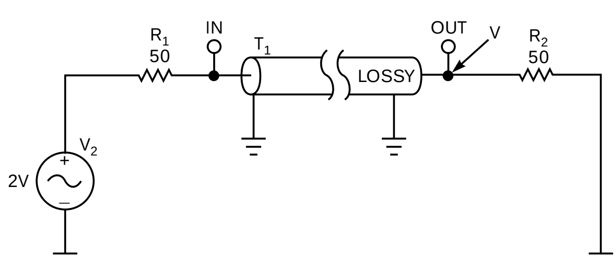

Figure 4 In the PSpice model for simulation, to match the VNA measurements, you must terminate the lossy line with a matched 50Ω termination, and the ac sine-wave source also must have 50Ω output impedance.

Figure 2 shows the measured loss curve as well as the model-simulation results. The match is excellent over the whole range, especially considering that the model comprises just a single circuit element in PSpice. The slight wiggle at high frequencies is due to reflections at the connector discontinuities, which the model does not include and therefore can't reproduce in the simulation. Figure 3 shows the parabolic fit and the resulting fit parameters. The dc resistance is negligible, and thus the curve fit omits it by setting the y intercept to zero. For the measured and fitted parameters, Z_0 is 50Ω, which you know and have verified with TDR; T_{PD} is 2.8 nsec, which you know and have verified using group delay or TDR; a is zero, with negligible dc losses; b is 2.826×10^{-5} from the curve fit; and d is 8.148×10^{-11}, also from the curve fit.

From this formula, you can calculate the model parameters for the lossy cable using equations 3, 12, and 15: LEN=1, C=56 pF, L=140 nH, R=$\{3.254\times10^{-4}\times\sqrt{\text{abs}(s/2\pi)}\}$, and L=$\{3.752\times10^{-13}\times\text{abs}(s/2\pi)\}$.

Figure 4 also displays these parameters along with the PSpice model used for simulation. To match the VNA measurements, you must terminate the lossy line with a matched 50Ω termination, and the ac sine-wave source also must have 50Ω impedance. Even though the plot is a frequency-domain sweep, you could just as easily use the same PSpice model for time-domain transient simulations without any changes to the model itself.

If you are interested in the quality of the cable's dielectric, you can determine the loss tangent using Equation 10 with C=56 pF and $k_{DIEL}=3.752\cdot10^{-13}$:

$$\tan\delta = \frac{k_{DIEL}}{2\pi\times C} = 0.0011, \qquad (16)$$

which shows that the dielectric is indeed a very-low-loss material.

If your Spice version offers no built-in lossy-transmission-line models, you can use these parameters for the whole line and divide them into a chain of small, lumped sections from discrete elements (Figure 1). However, the simulator must be able to accept frequency-dependent values for R and G, and this requirement will likely be a breaking point for many simulation tools. To avoid model artifacts, the propagation time through each lumped section should be smaller than about one-tenth of the fastest signal rise time for which the model shall be valid. In other words, the number of those sections should be at least

$$N \geq 10\times\frac{T_{PD}}{T_R}, \qquad (17)$$

and the component values for each section are:

$$C_I = \frac{C}{N},\ L_I = \frac{L}{N},\ R_I = \frac{R}{N},\ G_I = \frac{G}{N}. \qquad (18)$$

SUMMING UP

You can create highly usable, accurate models of lossy transmission lines, including cables and pc-board traces, using only a minimum of mathematics and readily available tools—a VNA, Excel, and PSpice—and following a simple recipe; you simultaneously gain understanding about loss behavior. An extension of the method would include approximate models of the connectors. A lumped capacitance or inductance often suffices, and you can analytically add it to the fitting process without recourse to numerical modeling, but you must experimentally determine the numerical values—either through TDR or time-domain transformation of the VNA data. Using the root-mean-square sum for skin resistance and dc resistance could also improve the quality of the fit parameters at the cost of making the fit nonpolynomial, which would require more elaborate fit routines. However, for most applications, the method is likely to provide more than adequate model accuracy and is easy to implement on a variety of fit and modeling tools.

Finally, many engineers working with digital signals have no access to a VNA but have oscilloscopes that can perform time-domain-transmission measurements; in such cases, you can harness step-transmission data to get the loss parameters through Fourier conversion of the time-domain data into S-parameters (Reference 4).EDN

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